

Robust Bias Mitigation for Localization in Wireless Networks

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By

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TABLE OF CONTENTS

	Page
List of Figures.....	vi
Abstract.....	vii
1. INTRODUCTION	1
2. BACKGROUND	5
2.1 Signal Propagation Model.....	5
2.2 Multi-iteration	10
2.2.1 Non-Linear Approach	11
2.2.2 Linear Approach	14
2.3 Related Work	18
3. LOCALIZATION ERROR MODELING	20
3.1 Error Distribution for Distance Estimates.....	20
3.2 Mean Square Error for the Location Estimate	23
3.2.1 Mean Square Error without the Bias.....	24
3.2.2 Mean Square Error in the Presence of Bias	25
3.2.3 Effect of Beacon Topology on the Mean Square Error	26
4. ERROR DECOMPOSITION AND ESTIMATION	30
4.1 Beacon Combinations	31
4.2 Location Error Decomposition and Estimation	32
5. PROPOSED DUAL WEIGHTED AVERAGE TECHNIQUE.....	35
5.1 Error Distribution Based on the Weighted Average Technique	35
5.2 Inverse Estimator Weighted Average (IEWA).....	37
5.3 Null Space Weighted Average (NSWA)	39
5.4 Final Location Estimate	41

6. RESULTS	42
6.1 Simulation Results	43
6.2 Experimental Results	45
7. CONCLUSION	46
List of References	48

LIST OF FIGURES

Figure	Page
Fig. 2.1 Path Loss Vs Distance	9
Fig. 2.2a. Distance Estimates without Error	10
Fig. 2.2b. Distance Estimates with Error	10
Fig. 2.3 Steepest Descent Algorithm	13
Fig. 2.4 Geometric Representation of the Linear Transformation.....	16
Fig. 2.5 Linear Transformation of System for $N = 3$	17
Fig. 2.6 Linear Transformation of System for $N = 4$	18
Fig. 3.1 MSE Comparison with one Biased Estimate (B1)	27
Fig. 3.2 MSE Comparison with two Biased Estimates (B1, B2).....	27
Fig. 3.3 MSE Comparison with all Biased Estimates (B4).....	28
Fig. 3.4 MSE Comparison for a Combination with Various Bias Configurations	28
Fig. 4.1 Location Estimates from Beacon Combinations	31
Fig. 4.2 Target Location Error Estimator.....	33
Fig. 5.1 Accuracy Improvement vs. Alpha	37
Fig. 6.1a, b, c Error vs. Number of Beacons.....	43
Fig. 6.2 CDF of Error for Various Bias Distributions	44
Fig. 6.3 Error Statistics from Experimental Testing	45

ABSTRACT

ROBUST BIAS MITIGATION FOR LOCALIZATION IN WIRELESS NETWORKS

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Localization of a wireless device or a sensor node has been a problem of great importance in recent years. Location services are becoming an essential part of the traditional wireless networks as well as the wireless sensor networks. Indoor localization in particular has been an issue of interest in recent studies since GPS is not suitable for indoor environments. The main issue in target node location estimation is the distance estimates affected by the non-line-of-sight (NLOS) signal measurements. In any practical setting, the NLOS errors are inevitable and need to be mitigated in order to achieve acceptable accuracy. In this work, first a theoretical analysis of effect of bias on the localization accuracy is performed. Then, a novel technique is presented which reduces the adverse effects of bias by utilizing the topological diversity provided by the L unique beacon combinations from the power set of $N(> 3)$ beacons. The proposed approach consists of two complementary weighted average methods combined to provide a high

level of robustness. The simulation and empirical results show that the presented technique is effective independent of the bias distribution. Thus it can be implemented on top of variety of existing localization methods in wireless networks to mitigate the adverse effects of NLOS propagation and achieve high accuracy in a practical scenario.

1. INTRODUCTION

Localization of a wireless device to estimate its location is a challenging problem. There has been a great amount of research on indoor localization, especially because of the fact that the “global positioning system” (GPS) is ineffective in an indoor environment. This necessitates the development of alternate approaches for solving the localization problem. The localization algorithms can be classified based on various criteria. They can be based on signal types (infrared, ultrasound, radio frequency), signal metrics (angle of arrival, time of arrival, received signal strength), and metric processing algorithms (triangulation, scene profiling). With the widespread of Wi-Fi technology, localization based on RF signal is a popular choice, compared to infrared and ultrasound localization. The latter techniques impose additional hardware requirements, and are beneficial only in certain special applications, such as Active Bat location system developed by AT&T researchers, or Cricket location support system developed by MIT computer science and artificial intelligence laboratory. Both of these systems use time-of-flight (TOF) data to compute the distance from the transmitter which requires synchronization. In general, algorithms using time of flight (TOF) or angle of arrival (AOA) as a signal metric need additional hardware for timing or beam forming purposes. AOA techniques require directional antennas which come at the cost of additional circuitry. In contrast, received signal strength (RSS) data is readily available in any Wi-Fi enabled device and no additional

hardware is required for computation. In addition, there is no need for synchronization. Nevertheless, the effectiveness of these algorithms highly depends on the environment as the properties of the wireless medium play a crucial role in the accuracy of each approach.

In an indoor facility, RF signal propagation is affected by a number of factors such as multi-path fading, temperature, humidity variations, and other dynamic factors including the presence and mobility of human beings. These factors make it difficult to define a mapping function from RF signal properties to distance between the transmitter and the receiver. One of the popular existing solutions to this problem is the “fingerprinting” approach, which circumvents the signal strength to distance transformation operation by directly linking a geographical location with a unique RSS signature. This approach requires an offline training stage which creates a database of these signatures for the entire facility. After deployment, the target is localized by matching the current RSS data with the signature database. Successful implementation of this technique is presented by the National Institute of Standards and Technologies (NIST) and by Ekahau on a commercial level as well. The major drawback of this approach is the offline calibration procedure. Creating and maintaining the RSS signature database is a cumbersome task, and has scalability issues. Hence they are useful in very specific applications.

In this work we mainly focus on online approaches based on RSS information. Low complexity of algorithms is a central theme of this paper, which facilitates actual testing,

rapid deployment and also low power requirements. We have assumed that RSS measurements are performed by the mobile sensor node itself; in contrast with the traditional network-based computational approach. All the simulation and experimental results are obtained using this approach. Nevertheless, it is possible to implement our algorithms in a centralized network-based system as well, since the algorithm is independent of system architecture. We also assume that the sensor node is provided with the topological information regarding the network prior to its deployment. The applications of such scenario include location identification for firefighters in emergency situations, for patients or visitors in hospitals, navigation assistance for the blind, and numerous commercial applications.

The thesis is organized as follows. Chapter 2 provides the background information regarding the different issues involved with RSS based localization, and how different approaches have addressed them. In a broad sense, mathematically the problem can be divided into two parts – first the modeling of wireless environment, which is a transformation from signal space to Euclidean space; and secondly, implementation of tri-lateration or a similar technique to transform from Euclidean distance space to Cartesian coordinates. Chapter 3 discusses the major issue involved in the localization in non-line-of sight environment. It talks about the signal propagation models, and the presence of “bias” in the distance estimates. It also provides mathematical analysis of the bias on the localization accuracy. Chapter 4 introduces the two key concepts used in the proposed approach to mitigate the effect of bias on localization accuracy. It talks about

the various beacon combinations as well as an error estimator to predict the reliability of the location estimation. Chapter 5 introduces the proposed algorithm which utilizes selected methods explained in chapter 3 and chapter 4 and modifies them in order to preserve low complexity and still achieve improved accuracy over existing approaches. It presents a new perspective for looking towards the localization problem using spring-force model concepts from physics. Based on these concepts it presents a novel dual weighted average algorithm which provided two fold robustness and high accuracy. Chapter 6 assesses the performance of the proposed algorithm and discusses complementary techniques which will assist the successful implementation in a practical scenario. The chapter also presents the results of the actual experiments performed in an indoor facility to verify the accuracy of the algorithm beyond simulation environment. Finally, we conclude by comparing the proposed algorithm with existing work from theoretical as well as practical stand point.

2. BACKGROUND

As mentioned earlier, online localization can be split into two sub-problems. First, it is important to model the wireless channel in order to estimate the distance information from the available RSS data. This problem is stochastic in nature as signal propagation is strongly affected by fading, shadowing etc. which introduces randomness. Especially in an indoor environment, the randomness is difficult to model with straightforward probability distributions, which makes this sub-problem quite challenging. The second sub-problem is estimation of the coordinates of the target location from the distance estimates. This part is purely mathematical, and several triangulation or tri-lateration methods are used to solve this problem. It is important to note that the error in the final location estimate is caused by randomness involved in the first sub-problem, as well as the unavoidable approximations made in the second sub-problem.

2.1 Signal Propagation Models

Signal propagation is an extremely complex phenomenon. The electromagnetic waves propagation undergoes scattering, diffraction, reflection due to the obstacles in the path. The popular models to estimate the signal propagation are the ray tracing techniques. The ray tracing techniques denote electromagnetic waves as simple particles. Even though ray tracing successfully models the effects of reflection and refraction, more complex

phenomenon such as scattering cannot be estimated by them. Maxwell's differential equations are more sophisticated techniques which explain scattering, but they are mostly ignored for all practical purposes. Besides the intrinsic complexity of propagation, the variability of the signal propagation channel or the environment makes it even more difficult to formulate deterministic models. Therefore, probabilistic models are used, and channels are defined by their statistical properties. Having this said, it is apparent that received signal power from the radio transmitter at a given distance is a stochastic process. Following is the review of popular empirical probabilistic models for signal propagation.

A. Okumura Model

Okumura Model is used for signal prediction in large urban areas. It is applicable for distances of 1-100 Km and frequency spectrum of 150-1500 MHz. The path loss formula is given as follows which is a function of distance, with frequency as a known parameter.

$$P_L(d)dB = L(f_c, d) + A_{mu}(f_c, d) - G(h_t) - G(h_r) - G_{area} \quad (2.1)$$

Where,

$L(f_c, d)$: free space path loss at distance d

$A_{mu}(f_c, d)$: median attenuation in addition to free path loss

$G(h_t)$: transmitter antenna gain

$G(h_r)$: receiver antenna gain

G_{area} : gain depending on type of environment

The Okumura model tries to capture the diversity of wireless channel by extensive experimental testing, and creating empirical tables or plots which categorize the values for A_{mu} and G_{area} .

B. Hata and Cost 231 Models

Hata model is the extension of Okumura model and it is valid for the same distance and the frequency ranges. It provides a closed form equation and gets rid of the empirical look up tables or plots. The parameters are the same as the Okumura model.

$$P_{L,urban}(d)dB = 69.55 + 26.16 \log_{10}(f_c) - 13.82 \log_{10}(h_t) - a(h_r) + (44.9 - 6.55 \log_{10}(h_t)) \log_{10}(d) \quad (2.2)$$

Cost 231 model developed by the European Cooperative for Scientific and Technical Research is fundamentally same model, but applicable to 2 GHz frequency band.

$$P_{L,urban}(d)dB = 46.3 + 33.9 \log_{10}(f_c) - 13.82 \log_{10}(h_t) - a(h_r) + (44.9 - 6.55 \log_{10}(h_t)) \log_{10}(d) + C_M \quad (2.3)$$

Where all the parameters are same as earlier two models except for C_M which is 0 dB for medium sized cities and 3 dB for metropolitan areas.

C. Indoor Models

Indoor environment presents additional attenuation factors which change the signal power drastically and quickly as well. Floors, walls, partitions, furniture and even human

beings affect signal power. The path loss suffered by the signal power for different materials is given in following table.

Table 2.1 Path Loss Values for Different Obstacles

Obstacle Type	Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete Wall	13
Aluminum Sliding	20.4
All Metal	26

The diversity and the random nature of the obstacles in the indoor environment makes it difficult to capture all the effects in one closed form formula. The popular approach is the use of simplified model which is a simple probabilistic approximation for the sum of all the attenuating factors in which the sum of attenuation due to various factors is represented as a shadowing parameter. The received power is then calculated with the following formula.

$$P_r \text{ dBm} = P_t \text{ dBm} - 10 \gamma \log_{10} \left(\frac{d}{d_0} \right) + \Psi \text{ dB} \quad (2.4)$$

where,

γ : Path loss exponent

d_0 : Reference distance, model valid for $d > d_0$

Ψ : Random variable representing lognormal shadowing

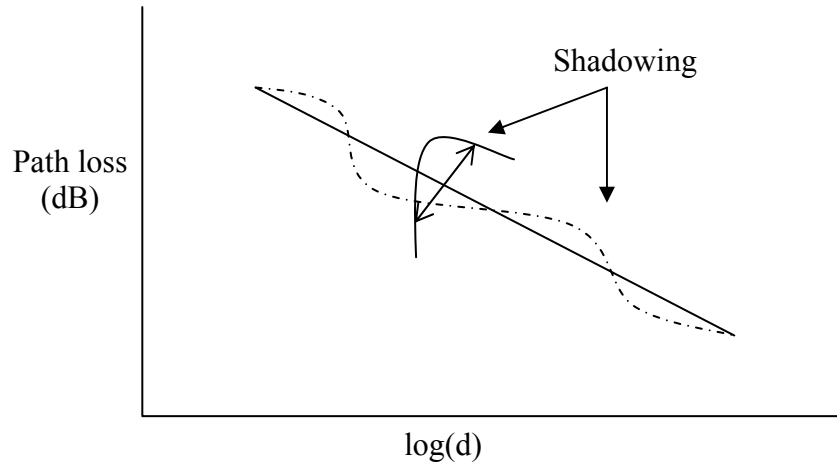


Fig. 2.1 Path Loss Vs Distance

The simplicity of this model has a great advantage for low complexity localization algorithms, and therefore this model is used throughout the work in this thesis. The distance is then calculated for measured receive power value by rearranging the equation as follows.

$$d = d_0 10^{\frac{P_t - P_r}{10\gamma}} * 10^{\frac{\psi}{10\gamma}} \quad (2.5)$$

Note that the term $10^{\frac{\psi}{10\gamma}}$ represents the error in the distance estimation, which depends on the random fluctuations in signal power denoted by ψ . We will discuss this in more detail in chapter 3.

2.2 Multi-Lateration

Multi-lateration is a fundamental technique which estimates the location from the available distance information from known reference points or beacons. For two-dimensional localization, theoretically three reference points are required, and for three-dimensional localization four reference points are needed. In this thesis, we only consider two-dimensional space; nevertheless, the algorithms are independent of this assumption. From a geometric point of view, the reference beacons generate circles (spheres in the case of three-dimensional space) with radii equal to estimated distances. In the case when distance estimates are free of error, then all circles intersect in one unique point which is the location of the target. In the presence of error, the circles will not intersect in a single point at the location of the target. This is shown in the figures below.

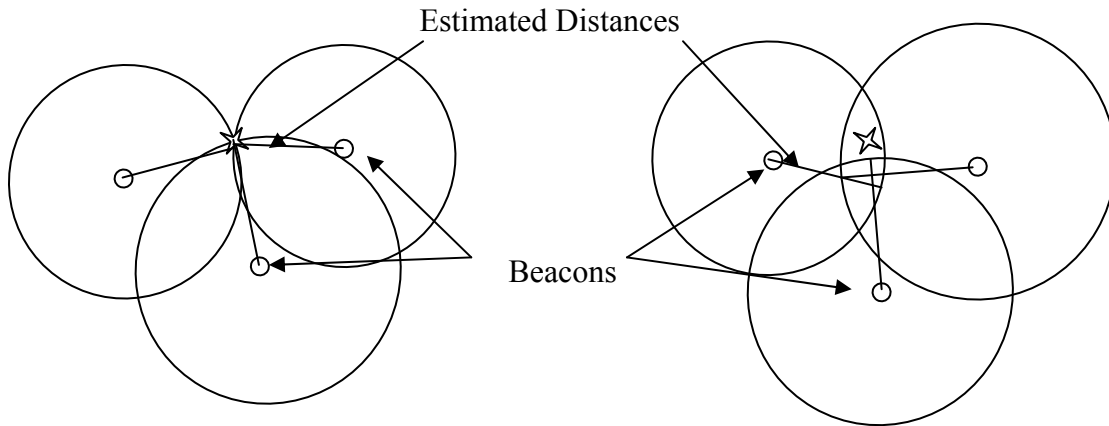


Fig.2.2 Distance Estimates (a) without Error (b) with Error

Analytically, the location of the target can be found by solving the equations of the circles. If there is no error present, the system of equations is consistent, and will yield a unique solution as long as distances from at least three beacons are known. However, when there is error present in the distance data, the equations will be inconsistent, and target location must be estimated using methods such as least squares. Following is a description of two multi-lateration approaches. The first uses a non-linear optimization technique and the other transforms the system into linear equations and solves using the linear least square technique.

2.2.1 Non-linear Approach

Assume that the number of reference points (beacons) is $N(\geq 3)$. Then following system of equations is obtained from distance estimates from each beacon.

$$(x - x_i)^2 + (y - y_i)^2 = \hat{d}_i^2, \quad i = 1, 2, \dots, N \quad (2.6)$$

$$\hat{d}_i = d_i + e_i \quad (2.7)$$

Where, (x_i, y_i) are the coordinates of the i^{th} beacon, (x, y) is the location of the mobile sensor node and \hat{d}_i is the distance estimate from i^{th} beacon. For nonzero error (e), these N equations are inconsistent. Then the estimation ($\hat{\mathbf{x}}$) for the target location is calculated as follows.

Denote,

$$\mathbf{x} = [x, y]$$

$$\mathbf{x}_i = [x_i, y_i]$$

$$\Rightarrow \|\mathbf{x} - \mathbf{x}_i\| = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (2.8)$$

Then,

$$\hat{\mathbf{x}} = \mathit{arg} \min_x \left\{ \sum_{i=1}^N (\hat{d}_i - \|\mathbf{x} - \mathbf{x}_i\|)^2 \right\} \quad (2.9)$$

In the case, where each equation is assigned a weight " w_i " based on some reliability criteria; then the above equation can be modified as follows.

$$\hat{\mathbf{x}} = \mathit{arg} \min_x \left\{ \sum_{i=1}^N w_i * (\hat{d}_i - \|\mathbf{x} - \mathbf{x}_i\|)^2 \right\} \quad (2.10)$$

In either case, multivariable optimization techniques must be used to estimate $\hat{\mathbf{x}}$. The popular approaches include Steepest Gradient Descent (SGD) and Gauss–Newton (GN) method. For the sake of analysis, we will present the SGD method and compare it with the linear approach used in this thesis.

Steepest Descent Algorithm

Define,

$$v_i = d_i - \|\mathbf{x} - \mathbf{x}_i\|$$

Then the objective function is expressed as,

$$F(\mathbf{x}) = \sum_{i=1}^N v_i^2$$

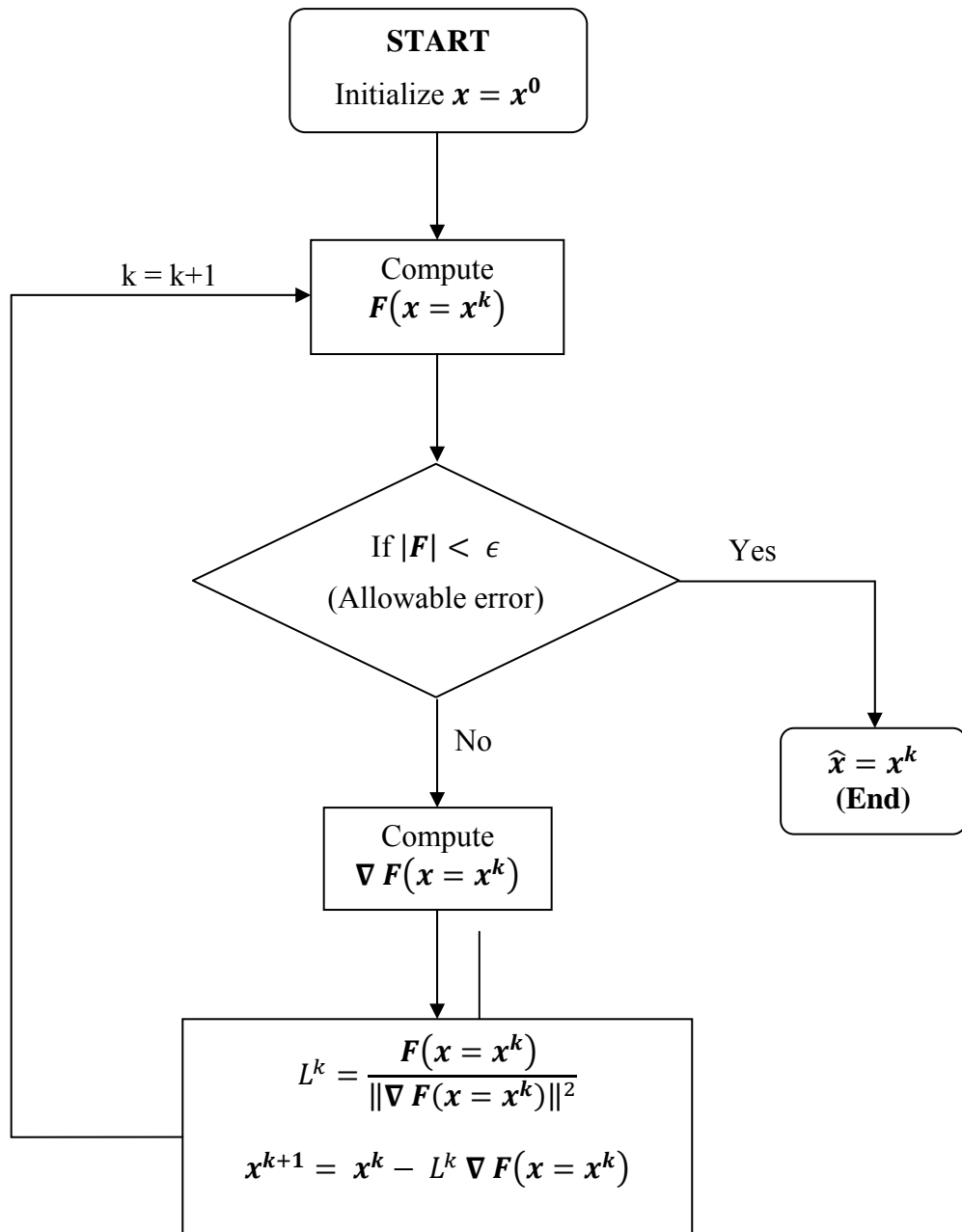


Figure 2.3 Steepest Descent Algorithm

The main drawback of the steepest descent method is that it requires an initial value – a rough estimate of the target location. Unless there is a good initial value estimate, the

algorithm will not converge to the correct local minimum. Depending on the localization application, initial location information is not always available. The second important issue regarding SGD is that the recursive nature of the method may take large number of iterations before reaching acceptable accuracy. Large number of iterations demands increased processing power, making it energy inefficient. To circumvent these difficulties, an alternate linear approach is used, which is described in the next section.

2.2.2 Linear Approach: Transformation into Linear System

From equation 2.6 we have the following set of N nonlinear equations.

$$\begin{aligned}
 (x - x_1)^2 + (y - y_1)^2 &= \hat{d}_1^2 \\
 (x - x_2)^2 + (y - y_2)^2 &= \hat{d}_2^2 \\
 &\vdots \\
 (x - x_N)^2 + (y - y_N)^2 &= \hat{d}_N^2
 \end{aligned} \tag{2.11}$$

Now these N equations can be transformed into N-1 linear equations by subtracting the r' th equation from remaining N-1 equations. This cancels out the quadratic x, y terms and following set of equations is obtained.

For $i = 1, 2, \dots, N$ ($i \neq r$)

$$\begin{aligned}
 (x_1 - x_r) x + (y_1 - y_r) y &= \frac{1}{2} [\hat{d}_r^2 - \hat{d}_1^2 - (x_r^2 + y_r^2) + (x_1^2 + y_1^2)] \\
 (x_2 - x_r) x + (y_2 - y_r) y &= \frac{1}{2} [\hat{d}_r^2 - \hat{d}_2^2 - (x_r^2 + y_r^2) + (x_2^2 + y_2^2)] \\
 &\vdots \\
 (x_N - x_r) x + (y_N - y_r) y &= \frac{1}{2} [\hat{d}_r^2 - \hat{d}_N^2 - (x_r^2 + y_r^2) + (x_N^2 + y_N^2)]
 \end{aligned} \tag{2.12}$$

which can be written in matrix form as,

For $i = 1, 2, \dots, N$ ($i \neq r$)

$$\begin{bmatrix} x_1 - x_r & y_1 - y_r \\ x_2 - x_r & y_2 - y_r \\ \vdots & \vdots \\ x_N - x_r & y_N - y_r \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \hat{d}_r^2 - \hat{d}_1^2 - (x_r^2 + y_r^2) + (x_1^2 + y_1^2) \\ \hat{d}_r^2 - \hat{d}_2^2 - (x_r^2 + y_r^2) + (x_2^2 + y_2^2) \\ \vdots \\ \hat{d}_r^2 - \hat{d}_N^2 - (x_r^2 + y_r^2) + (x_N^2 + y_N^2) \end{bmatrix} \quad (2.13)$$

Now this system can be represented as: $\mathbf{Ax} = \frac{1}{2} \mathbf{B}$ and can be solved using the least squares method.

$$\hat{\mathbf{x}} = \frac{1}{2} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{B} \quad (2.14)$$

Geometric Interpretation of Nonlinear to Linear Transformation

By transforming the system from non-linear to linear, we are no longer solving the equations of the circles; instead, we are solving the equations of the lines passing through the intersection points of the overlapping circles. In case the circles do not intersect with each other, the transformation still generates a line which is perpendicular to the line joining the two centers of the circles. The following figures show the three possible scenarios.

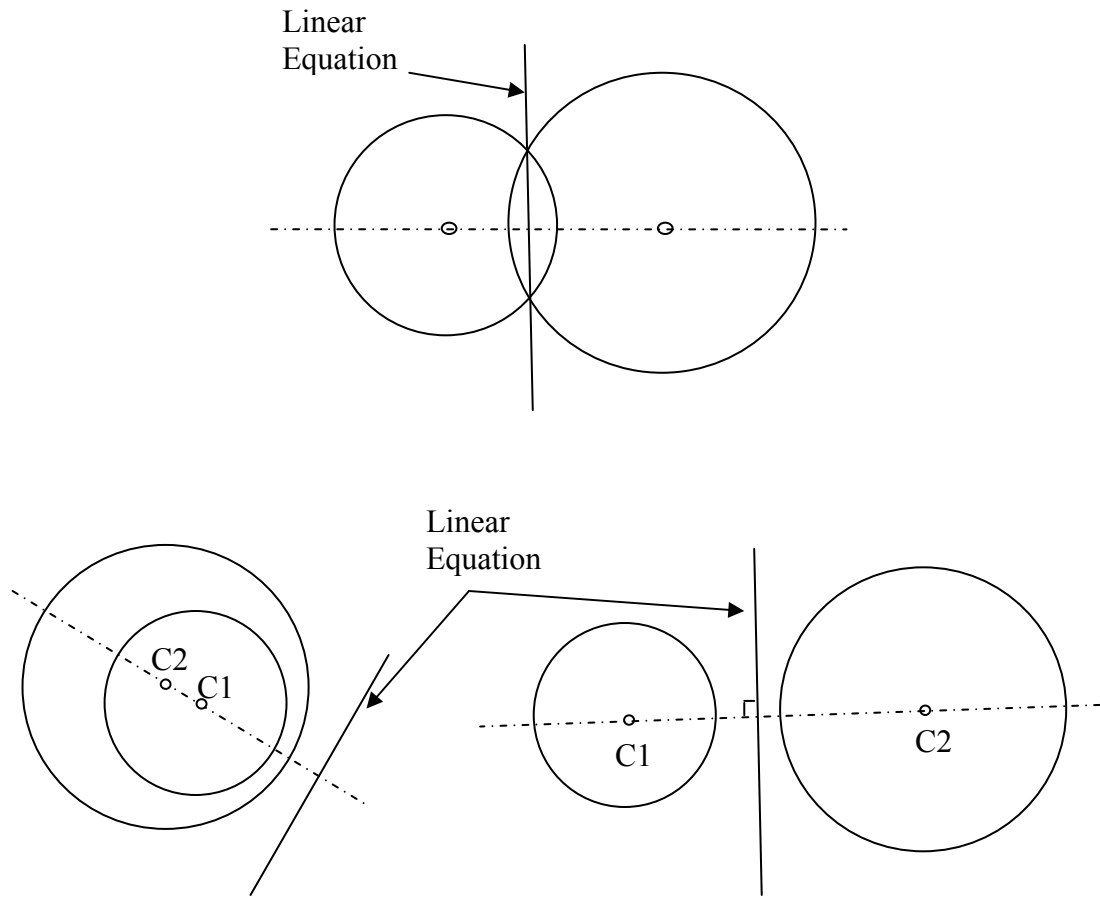


Fig. 2.4 Geometric Representation of the Linear Transformation

It is important to note that, depending on the choice for the reference beacon (r' th equation) different sets of lines are generated. For instance, in the Fig. 2.5 if the reference beacon selected is A then lines L1 and L2 will be generated and not L3. And the least square solution will be the point P which is an intersection of lines L1 and L2. Interestingly, line L3 will also intersect the other two lines at point P. This will always be the case for N equal to three. Consequently, the selection of reference beacon does not influence the final target estimation. However, this is not the case for $N > 3$. As shown in

the Fig 2.6 different combinations of circles will yield different sets of lines which will not intersect in at unique point. These different systems of linear, but inconsistent equations are solved using least squares method, however, the final target location is no longer a unique solution. It strongly depends on the choice of the reference beacon. For instance, if B is selected as the reference beacon, then lines L1, L3 and L4 are generated. Whereas, if C is selected as the reference beacon, then lines L2,L3 and L5 are generated, which will have a different least square solution compared to the previous combination. We will revisit this issue in chapter 4 when we discuss the effect of bias in distance estimates.

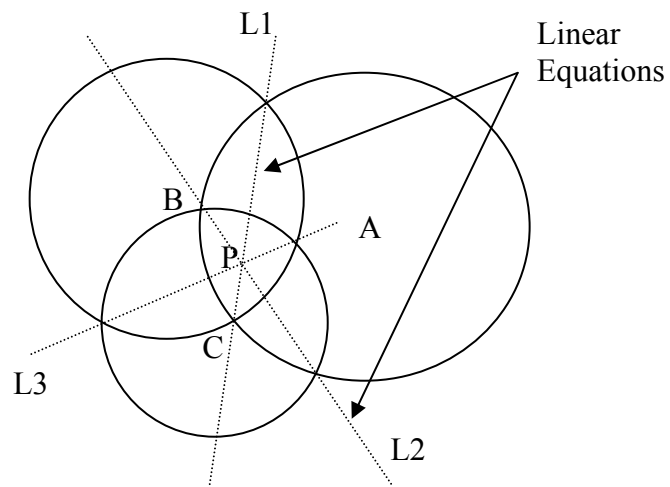


Fig. 2.5 Linear Transformation of System for $N = 3$

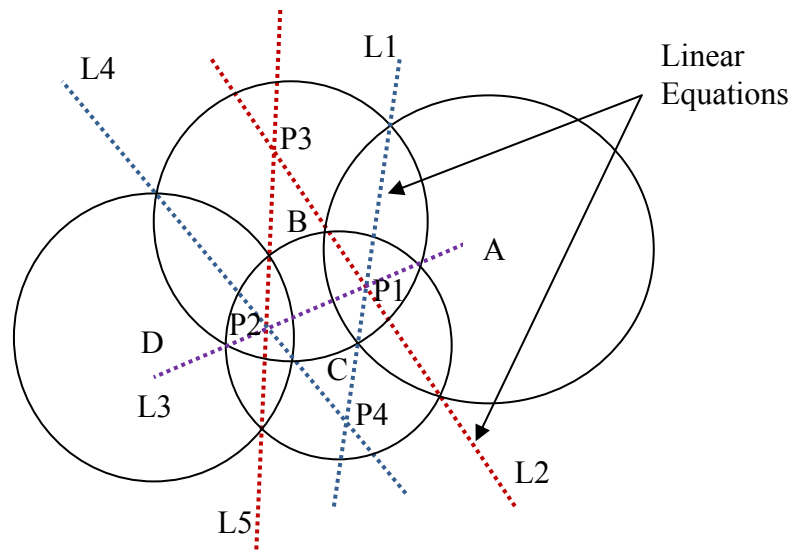


Fig. 2.6 Linear Transformation of System for $N = 4$

2.3 Related Work

Several methods have been proposed to model and mitigate the effect of the error in the distance estimates. This error is a result of the biased distance estimates from NLOS signal propagation. The bias can be modeled with different probability distributions such as Exponential [1], Uniform [2] and Gaussian [3]. It is assumed to be a positive quantity in virtually all scenarios since attenuation of signal is more likely than amplification. The effect of bias on the location estimate is arbitrary and depends on multiple factors, such as topology of the beacons, location of the target node as well as the localization technique. One class of bias mitigation techniques includes identification and rejection of the NLOS distance estimates [4] [5]. A major drawback of this approach is the requirement of at least three LOS or unbiased measurements, which may not be possible

in many practical cases. Other methods of NLOS error mitigation techniques include addition of a correction factor [6], linear programming [3] and weighted average [1]. Most of these approaches are dependent on the underlying assumption for the bias distribution and thus they are only effective for specific applications. From these various techniques, we will use the Rwgs algorithm proposed in [1] for comparison purposes, as it uses a similar weighted average techniques utilized in this work.

3. LOCALIZATION ERROR MODELING

In this chapter, we will discuss the various factors that generate the error in the location estimate, and different statistical approaches of modeling them. As mentioned earlier, RSS based localization is a two step process. First step is the transformation from signal space to Euclidean distance space. In this step, the errors are caused due to the complexity of signal propagation and the inevitable randomness associated with it. This randomness is the major source of error in the distance estimates which translates into the error in the location estimate in Cartesian space. However, it is important to note that the error in the location estimate is not linearly related with the error in the distance estimates. This fact is elaborated more in detail in chapter 4. But first, following is the description of the distance error model.

3.1 Distance Error Estimation Using Simplified Path Loss Model

$$P_r \text{ dBm} = P_t \text{ dBm} - 10 \gamma \log_{10} \left(\frac{\hat{d}}{d_0} \right) + \Psi \text{ dB} \quad (3.1)$$

where $\Psi \sim N(0, \sigma^2)$

By rearranging the above equation we get,

$$\begin{aligned} \hat{d} &= d_0 10^{\frac{P_t - P_r}{10 \gamma}} * 10^{\frac{\Psi}{10 \gamma}} \\ &= d_0 e^{\frac{2.303(P_t - P_r)}{10 \gamma}} * e^{\frac{2.303 \Psi}{10 \gamma}} \end{aligned}$$

$$\Rightarrow \hat{d} = d * e^{\frac{2.303\psi}{10\gamma}}$$

$$\Rightarrow \hat{d} = d * e^{\lambda}$$

Where d is the actual distance between the transmitter and the receiver and $\lambda = \frac{2.303\Psi}{10*\gamma}$

From equation 2.7,

$$e = \hat{d} - d$$

$$\Rightarrow e = d * e^{\lambda} - d = d (e^{\lambda} - 1)$$

Now using the Taylor series expansion,

$$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2!} \dots \approx 1 + \lambda$$

Since $\lambda < 1$ for 99% of times for $\Psi \sim N(0, \sigma^2)$ with $\sigma^2 \geq 6\text{dB}$ and $\gamma \geq 4$

$$\Rightarrow e = d * \lambda = d * \frac{2.303\Psi}{10*\gamma}$$

$$\Rightarrow e \sim N(0, d^2\sigma_{\lambda}^2) \tag{3.2}$$

For $\Psi \sim N(0, 6\text{dB})$ and $\gamma = 4$

$$\sigma_{\lambda} = 0.345$$

$$\Rightarrow e \sim N(0, (0.345d)^2) \tag{3.3}$$

The above model for error in distance estimation gives us a basic idea regarding the relationship between signal propagation and the distance. The model shows us that the variance of the error is strongly correlated with the distance between the transmitter and the receiver, which itself is an unknown parameter. Nevertheless, the assumption that signal propagation only consists of shadowing (Ψ) random variable is far from practical situation, particularly in an indoor environment. If distance error is purely normally

distributed random variable with zero mean, then the error could be reduced significantly by averaging a high number of distance estimation samples. However, this is not the case in practical conditions. Therefore, more complex models are needed for accurate realization of the system. Following is a popular model for distance estimation, which accommodates two types of signal propagation scenarios – line of sight (LOS) and non line of sight (NLOS).

$$\hat{d} = d + n + b \quad (3.4)$$

where,

$$n \sim N(0, \sigma^2)$$

b ~ Uniform , Exponential , Normal distribution

The model assumes that signal propagation from a LOS path will only contribute errors with zero mean normal distribution, but the errors caused due to signal propagation from NLOS path need to be modeled differently. The NLOS path errors are often called as the “bias” in the distance estimation. The bias can be modeled with a uniform, exponential, Gaussian or some other distribution depending on the environment. In popular approaches, the bias is often modeled as a positive random variable. This assumption is based on the fact that attenuation of the signal is more likely than the amplification, which results into overestimation of the distance. We will use this model as a basis to calculate the error in location estimation from the distance data, which is described in the next section.

3.2 Error in the Location Estimate

In this section, we will show the derivation for the mean square error (MSE) in the location estimate, and discuss the factors that influence the error. We will use the linear least squares method to calculate the location estimate described in chapter 3. From equation 2.14 we have,

$$\hat{\mathbf{x}} = \frac{1}{2} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{B} \quad (3.5)$$

where

$$\mathbf{B} = \begin{bmatrix} \hat{d}_r^2 - \hat{d}_1^2 - (x_r^2 + y_r^2) + (x_1^2 + y_1^2) \\ \hat{d}_r^2 - \hat{d}_2^2 - (x_r^2 + y_r^2) + (x_2^2 + y_2^2) \\ \vdots \\ \hat{d}_r^2 - \hat{d}_N^2 - (x_r^2 + y_r^2) + (x_N^2 + y_N^2) \end{bmatrix}$$

The matrix \mathbf{B} can be decomposed into actual and error terms using Eq. (2.7) as follows:

$$\mathbf{B} = \mathbf{B}_a + \mathbf{B}_e$$

$$\mathbf{B}_a = \begin{bmatrix} d_r^2 - d_1^2 - (x_r^2 + y_r^2) + (x_1^2 + y_1^2) \\ d_r^2 - d_2^2 - (x_r^2 + y_r^2) + (x_2^2 + y_2^2) \\ \vdots \\ d_r^2 - d_N^2 - (x_r^2 + y_r^2) + (x_N^2 + y_N^2) \end{bmatrix} \quad (3.6)$$

$$\mathbf{B}_e = \begin{bmatrix} 2d_r e_r - 2d_1 e_1 + e_r^2 - e_1^2 \\ 2d_r e_r - 2d_2 e_2 + e_r^2 - e_2^2 \\ \vdots \\ 2d_r e_r - 2d_N e_N + e_r^2 - e_N^2 \end{bmatrix} \quad (3.7)$$

Then, the covariance of $\hat{\mathbf{x}}$ can be calculated as follows,

$$\text{Cov}(\hat{\mathbf{x}}) = E\{(\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^T\}$$

where $\mathbf{x} = (x, y)$ is the actual location of the target, which can be denoted as,

$$\mathbf{x} = \frac{1}{2} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{B}_a \quad (3.8)$$

Therefore

$$\begin{aligned}
Cov(\hat{x}) &= \frac{1}{4} E\{((A^T A)^{-1} A^T B - (A^T A)^{-1} A^T B_a)((A^T A)^{-1} A^T B - (A^T A)^{-1} A^T B_a)^T\} \\
&= \frac{1}{4} (A^T A)^{-1} A^T E\{B_e B_e^T\} A (A^T A)^{-1}
\end{aligned} \tag{3.9}$$

and

$$MSE = Trace(Cov(\hat{x})) \tag{3.10}$$

$E\{B_e B_e^T\}$ is an $N-1$ by $N-1$ matrix, and it strongly depends on the error model used. Two closed formed solutions have been derived in two scenarios which give us the fundamental platform and we use it as a basis for our proposed algorithm. Here we provide a brief derivation of the MSE, based on the paper by Guvenc, Chong and Watanabe.

3.2.1 Mean Square Error without the Bias (Case I)

$$\hat{d} = d + n$$

$$\Leftrightarrow e = n$$

$$\begin{aligned}
E\{B_e B_e^T\}_{ij} &= \{(2d_r n_r - 2d_i n_i + n_r^2 - n_i^2) \times (2d_r n_r - 2d_j n_j + n_r^2 - n_j^2)\} \\
&= E\{4d_r^2 n_r^2 + n_r^4 - n_r^2 n_j^2 - n_r^2 n_i^2 + n_i^2 n_j^2\} \\
&= 4d_r^2 \sigma^2 + 6\sigma^4; \quad (i \neq j)
\end{aligned} \tag{3.11}$$

and

$$\begin{aligned}
E\{B_e B_e^T\}_{ii} &= \{(2d_r n_r - 2d_i n_i + n_r^2 - n_i^2)^2\} \\
&= E\{4d_r^2 n_r^2 + 4d_i^2 n_i^2 + n_r^4 - n_r^2 n_j^2 - n_r^2 n_i^2 + n_i^4\} \\
&= 4\sigma^2(d_r^2 + d_i^2) + 8\sigma^4
\end{aligned} \tag{3.12}$$

3.2.2 Mean Square Error in the Presence of Bias (Case II)

$$\hat{d} = d + n + b$$

To distinguish from the previous case we will denote the matrix with error terms by $\tilde{\mathbf{B}}_e$ instead of \mathbf{B}_e which did not have bias terms in it.

Now

$$\tilde{\mathbf{B}}_e = \begin{bmatrix} 2d_r e_r - 2d_1 e_1 + e_r^2 - e_1^2 \\ 2d_r e_r - 2d_2 e_2 + e_r^2 - e_2^2 \\ \vdots \\ 2d_r e_r - 2d_N e_N + e_r^2 - e_N^2 \end{bmatrix}$$

Where error is decomposed as: $e = n + b$

$$\tilde{\mathbf{B}}_e = \begin{bmatrix} \tilde{d}_1 + \tilde{n}_1 + \tilde{b}_1 + \tilde{c}_1 \\ \tilde{d}_2 + \tilde{n}_2 + \tilde{b}_2 + \tilde{c}_2 \\ \vdots \\ \tilde{d}_N + \tilde{n}_N + \tilde{b}_N + \tilde{c}_N \end{bmatrix}$$

where

$$\tilde{d}_i = 2(d_r n_r - d_i n_i)$$

$$\tilde{n}_i = n_r^2 - n_i^2$$

$$\tilde{b}_i = b_r^2 - b_i^2$$

$$\tilde{c}_i = 2(d_r b_r - d_i b_i + b_r n_r - b_i n_i)$$

Therefore

$$\tilde{\mathbf{B}}_e = \mathbf{B}_e + \tilde{\mathbf{b}} + \tilde{\mathbf{c}}$$

and the covariance can be calculated as follows,

$$\begin{aligned} \text{Cov}(\hat{\mathbf{x}}) &= \frac{1}{4} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T E \{ \tilde{\mathbf{B}}_e \tilde{\mathbf{B}}_e^T \} \mathbf{A} (\mathbf{A}^T \mathbf{A})^{-1} \\ E \{ \tilde{\mathbf{B}}_e \tilde{\mathbf{B}}_e^T \} &= E \{ \mathbf{B}_e \mathbf{B}_e^T \} + E \{ \mathbf{B}_e \tilde{\mathbf{b}}^T \} + E \{ \mathbf{B}_e \tilde{\mathbf{c}}^T \} + E \{ \tilde{\mathbf{b}} \mathbf{B}_e^T \} + E \{ \tilde{\mathbf{b}} \tilde{\mathbf{b}}^T \} + E \{ \tilde{\mathbf{b}} \tilde{\mathbf{c}}^T \} + \\ &\quad E \{ \tilde{\mathbf{c}} \mathbf{B}_e^T \} + E \{ \tilde{\mathbf{c}} \tilde{\mathbf{b}}^T \} + E \{ \tilde{\mathbf{c}} \tilde{\mathbf{c}}^T \} \end{aligned} \quad (3.13)$$

where

$E\{\mathbf{B}_e \mathbf{B}_e^T\}$ is given by equations 3.11 and 3.12

$$E\{\tilde{\mathbf{b}}\tilde{\mathbf{b}}^T\}_{ij} = (b_r^2 - b_i^2) \times (b_r^2 - b_j^2)$$

$$E\{\tilde{\mathbf{c}}\tilde{\mathbf{c}}^T\}_{ij} = 4(d_r b_r - d_i b_i)(d_r b_r - d_j b_j) + 4\sigma^2 b_r^2 + I(i, j)4\sigma^2 b_i^2$$

$$E\{\mathbf{B}_e \tilde{\mathbf{b}}^T\}_{ij} = 2\sigma^2(b_i^2 + b_r^2) = E\{\tilde{\mathbf{b}}\mathbf{B}_e^T\}_{ji}$$

$$E\{\mathbf{B}_e \tilde{\mathbf{c}}^T\}_{ij} = 8d_r b_r \sigma^2 + 4d_j b_j \sigma^2 + I(i, j)4d_i b_i \sigma^2 = E\{\tilde{\mathbf{c}}\mathbf{B}_e^T\}_{ji}$$

$$E\{\tilde{\mathbf{b}}\tilde{\mathbf{c}}^T\}_{ij} = 2(b_r^2 - b_i^2)(d_r b_r - d_j b_j) = E\{\tilde{\mathbf{c}}\tilde{\mathbf{b}}^T\}_{ji}$$

Where,

$$\begin{aligned} I(i, j) &= 1; \text{ if } i = j \\ &= 0; \text{ if } i \neq j \end{aligned}$$

3.2.3 Effect of Beacon Topology on the Mean Square Error

To understand the effect of bias on location error we performed simulations using the above set of equations for different beacon combinations. For the following results, beacons ($B_i, i = 1:4$) are located at $[0, 0]$, $[40, 0]$, $[40, 40]$ and $[0, 40]$. The path of the target node is along the diagonal from B_1 to B_3 with an increment of one unit in both x and y coordinates. Since the increments are identical, the location index refers to both the coordinates of the target node. The noise component is normally distributed as: $\mathcal{N}(0, 1)$. Bias is 2 for Fig. 3.1, Fig 3.2 and Fig. 3.4 and uniformly distributed from $[0, 2]$ for Fig. 3.3. In the Fig. 3.1 only B_1 provides biased distance estimate. Fig. 3.2 shows the effect of two biased estimates and Fig. 3.3 shows when all estimates are biased with a random

amount. Fig. 3.4 shows the effect of different bias configurations on the location estimation computed from a particular beacon combination.

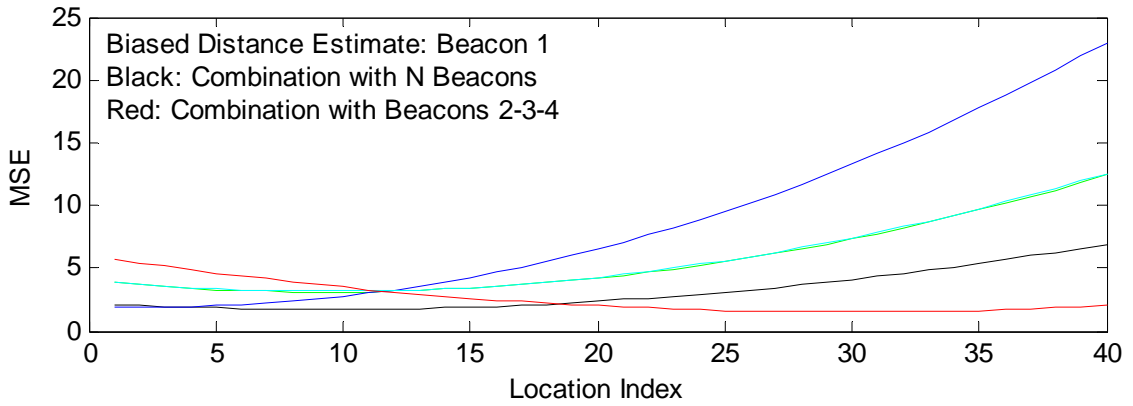


Fig.3.1 MSE Comparison – One Biased Estimate

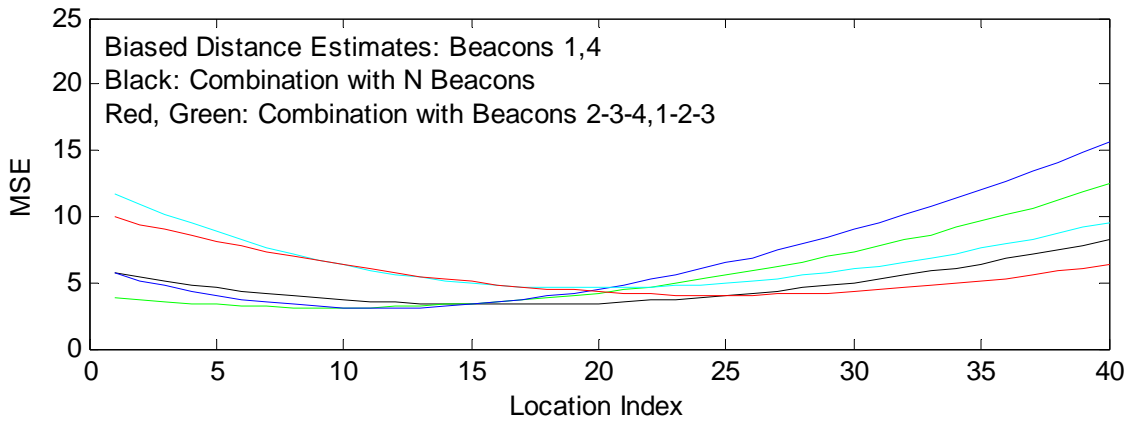


Fig.3.2 MSE Comparison – Two Biased Estimates

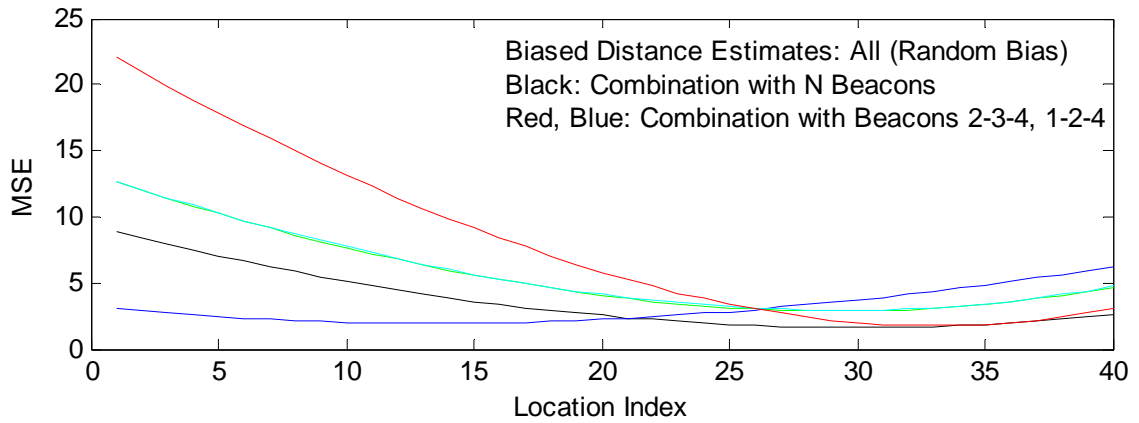


Fig.3.3 MSE Comparison – All Biased Estimates (Random Bias)

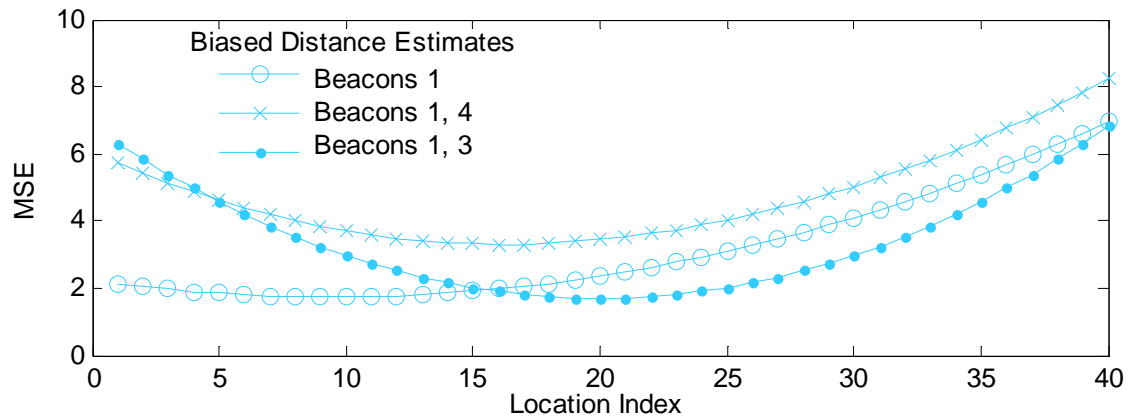


Fig. 3.4 MSE Comparison for Particular Beacon Combination with Various Bias Configurations

The observations made from these figures are as follows.

1. The effect of bias is highly dependent on the target location, and exclusive use of combination with all (N) beacons will not provide optimal results. Some combinations will suffer more than others from the accuracy perspective.

2. The topology of beacons plays an important role. It is possible to have a combination with high amount of bias and still provide a better location estimate than a combination with relatively small amount of bias.
3. Lastly, as a corollary of the first two observations, it can be seen that symmetry can help reduce the effect of bias. Fig.3.4 shows that the MSE in the case of symmetrical biased beacons (B_1, B_3) is lower than the asymmetric case (B_1, B_4) . Interestingly, at some locations, it is even lower than the MSE in the case of the combination with single biased beacon (B_1) .

Based on these observations we define an estimator that is dependent on the beacon topology and captures the effect of symmetry. Hence it has a strong correlation with the error. This estimator is described in the next chapter.

4. ERROR DECOMPOSITION AND ESTIMATION

In this chapter we explain a novel algorithm based on the observations discussed previously. The primary goal of the algorithm is to reduce the effect of biased distance estimates on the target location error. The algorithm is divided in two parts. First part deals with combinatorics of the beacons, in which all possible combinations of three or more beacons are considered for location computation. The second part consists of the selection process of identifying the “good” or more reliable combinations versus the “bad” or unreliable combinations. We discuss four possible selection methods based on a “spring-mechanics” model. We show through simulation results that the algorithm is independent of statistical distribution of the bias component. We also provide the experimental results to validate the simulation results and show that the algorithm reduces errors and provides higher accuracy over existing techniques.

4.1 Beacon Combinations

Let S be the set of N beacons. Then the set of all possible combinations of N beacons is the power set of S . Then number of total combination is given by,

$$|\wp(S)| = 2^N$$

where $\wp(S)$ is the power set of S .

However, since we are considering the combinations of 3 beacons or more, we need to subtract the combinations of two or smaller number of beacons from the power set. Then the resultant set is the subset of the power set, and the number of elements in the subset is given by,

$$\begin{aligned}
 L &= |\wp^*(S)| = 2^N - \binom{N}{2} - \binom{N}{1} - \binom{N}{0} \\
 &= 2^N - \frac{N^2 + N + 2}{2}
 \end{aligned} \tag{4.1}$$

Hence L increases exponentially as a function of N . Fig.4.1 shows the L location estimates linked with the L node combinations.

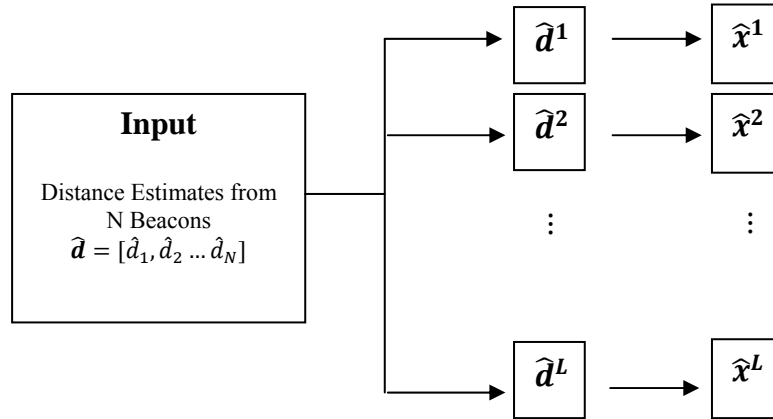


Fig.4.1 Location Estimates from Beacon Combinations

where

$$\hat{\mathbf{a}}^1 = [\hat{d}_1, \hat{d}_2, \hat{d}_3]$$

$$\hat{\mathbf{a}}^2 = [\hat{d}_1, \hat{d}_2, \hat{d}_4]$$

⋮

$$\hat{\mathbf{a}}^L = [\hat{d}_1, \hat{d}_2 \dots \hat{d}_N]$$

$$\hat{\mathbf{x}}^i = \frac{1}{2} (A^{iT} A^i)^{-1} A^{iT} B^i$$

$\hat{\mathbf{x}}^i$ is the i^{th} location estimate associated with the corresponding distance estimates. These L location estimates provide more diversity as some beacon combinations will provide more accurate location estimation than the others. As discussed in the previous chapter, depending on the amount of bias, topology of the beacons and few other factors, lower order combinations can provide higher accuracy compared to the location estimate from the higher order system. Now, it is crucial to identify these more accurate combinations from the others in order to improve the accuracy. The next section discusses these selection processes and compares them.

4.2 Location Error Decomposition and Estimation

Let $\hat{\mathbf{x}}$ be the target location estimate from some combination of M beacons. Then we define a vector estimator,

$$\mathbf{\Gamma} = \sum_{j=1}^M \frac{\hat{d}_j - \|\hat{\mathbf{x}} - \mathbf{x}_j\|}{\|\hat{\mathbf{x}} - \mathbf{x}_j\|} (\hat{\mathbf{x}} - \mathbf{x}_j) \quad (4.2)$$

where,

\mathbf{x}_j : j^{th} Beacon location ($j = 1, 2, \dots, M$)

\hat{d}_j = distance estimate from j^{th} beacon from signal measurement (RSS, TOA)

The geometric representation of this vector estimator is given in Fig. 4.2. The red arrow represents the location error estimator which is the vector sum of the residual distance estimates, i.e. $\hat{d}_j - \|\hat{\mathbf{x}} - \mathbf{x}_j\|$ represented by the blue arrows.

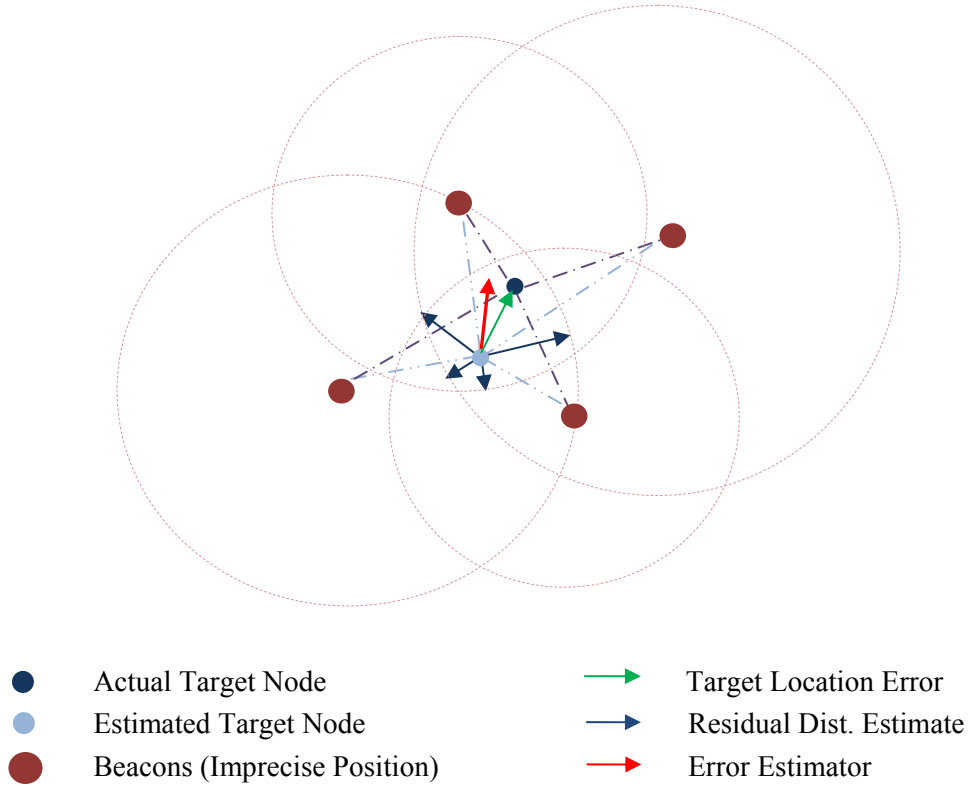


Fig.4.2 Target Location Error Estimator

The error estimator can be written in terms of the actual error vector as follow.

Denote

$$r_j = \frac{\hat{d}_j - \|\hat{\mathbf{x}} - \mathbf{x}_j\|}{\|\hat{\mathbf{x}} - \mathbf{x}_j\|} \quad (4.3)$$

and substitute $\hat{\mathbf{x}} = \mathbf{x} + \boldsymbol{\xi}$

$$\begin{aligned}\boldsymbol{\Gamma} &= \sum_{j=1}^M r_j (\mathbf{x} + \boldsymbol{\xi} - \mathbf{x}_j) \\ \boldsymbol{\xi} &= \frac{\boldsymbol{\Gamma} + \sum_{j=1}^M r_j (\mathbf{x}_j - \mathbf{x})}{\sum_{j=1}^M r_j} \\ \boldsymbol{\xi} &= a\boldsymbol{\Gamma} + \mathbf{G}\end{aligned}\tag{4.4}$$

where, $\boldsymbol{\xi}$ is the error in the location estimate. And,

$$\begin{aligned}a &= \frac{1}{\sum_{j=1}^M r_j} \\ \mathbf{G} &= \frac{\sum_{j=1}^M r_j (\mathbf{x}_j - \mathbf{x})}{\sum_{j=1}^M r_j},\end{aligned}\tag{4.5}$$

Now for any given beacon combination, location estimate error can be decomposed into two vector components, $a\boldsymbol{\Gamma}$ and \mathbf{G} . Note that \mathbf{G} is a weighted vector sum of actual distances from beacons to target node. We assume the x and y components of \mathbf{G} for any beacon combination as follows,

For i 'th combination ($i = 1, 2, \dots, L$)

$$G_{i_x} \sim \mathcal{N}(\mu_{G_{i_x}}, \sigma^2) \text{ and } G_{i_y} \sim \mathcal{N}(\mu_{G_{i_y}}, \sigma^2)\tag{4.6}$$

5. PROPOSED DUAL WEIGHTED AVERAGE TECHNIQUE

5.1 Error Distribution Based on the Weighted Average Technique

Let $\tilde{\mathbf{x}}$ be the location estimate from a weighted average of L initial location estimates.

Then

$$\tilde{\mathbf{x}} = \sum_{i=1}^L w_i \hat{\mathbf{x}}_i \quad (5.1)$$

where, w_i is the weight of the i 'th combination.

and $\sum w_i = 1$.

Then the error vector for the final location can be computed as follows.

$$\tilde{\boldsymbol{\xi}} = \tilde{\mathbf{x}} - \mathbf{x} = \sum_{i=1}^L w_i \hat{\mathbf{x}}_i - \sum_{i=1}^L w_i \mathbf{x}$$

$$\tilde{\boldsymbol{\xi}} = \sum_{i=1}^L w_i \boldsymbol{\xi}_i$$

$$\boldsymbol{\xi}_i = a_i \boldsymbol{\Gamma}_i + \mathbf{G}_i$$

Therefore from (4.6),

$$\boldsymbol{\xi}_i \sim \mathcal{N}(a_i \boldsymbol{\Gamma}_i + \boldsymbol{\mu}_{G_i}, \boldsymbol{\Sigma}) \quad (5.2)$$

$$\tilde{\boldsymbol{\xi}} \sim \mathcal{N}\left(\sum_{i=1}^L w_i (a_i \boldsymbol{\Gamma}_i + \boldsymbol{\mu}_{G_i}), \tilde{\boldsymbol{\Sigma}}\right) \quad (5.3)$$

and

$$\|\xi_i\| \sim \text{Rice}(v_i, \sigma) \quad (5.4)$$

$$\|\tilde{\xi}\| \sim \text{Rice}(\tilde{v}, \tilde{\sigma}) \quad (5.5)$$

where

$$\boldsymbol{\mu}_{Gi} = [\mu_{Gi_x}, \mu_{Gi_y}]$$

$$\tilde{\sigma}^2 = \sum_{i=1}^L w_i^2 \sigma^2 < \sum_{i=1}^L w_i \sigma^2 = \sigma^2 \quad (\text{Assuming } \xi_i \text{ are independent})$$

$\boldsymbol{\Sigma}$ = Covariance Matrix of ξ_i

$\tilde{\boldsymbol{\Sigma}}$ = Covariance matrix of $\tilde{\xi}$

$$v_i = \|a_i \boldsymbol{\Gamma}_i + \boldsymbol{\mu}_{Gi}\|$$

$$\tilde{v} = \|\sum_{i=1}^L w_i (a_i \boldsymbol{\Gamma}_i + \boldsymbol{\mu}_{Gi})\|$$

Now

$$E\{\|\tilde{\xi}\|\} < E\{\|\xi_i\|\}$$

if

$$\tilde{\sigma} \sqrt{\frac{\pi}{2}} L_{1/2} \left(-\frac{\tilde{v}^2}{2\tilde{\sigma}^2} \right) < \sigma \sqrt{\frac{\pi}{2}} L_{1/2} \left(-\frac{v_i^2}{2\sigma^2} \right) \quad (5.6)$$

where the parameters are as follows.

$$L_{1/2}(t) = e^{t/2} [(1-t)I_0 \left(-\frac{t}{2} \right) - t I_1 \left(-\frac{t}{2} \right)]$$

I_0, I_1 : Bessel Functions

Let

$$\alpha = \frac{v_i}{\tilde{v}}, \beta = \frac{\sigma^2}{\tilde{\sigma}^2} \text{ and } \gamma = \frac{E\{\|\mathbf{e}_i\|\}}{E\{\|\tilde{\mathbf{e}}\|\}} \quad (5.7)$$

Then Fig.5.1 shows the interrelationship between these three parameters. Note that, $\gamma > 1$ implies that there is an improvement in the location accuracy. Since, $\tilde{\sigma}^2 < \sigma^2$ therefore, $\beta > 1$. Thus the only crucial criterion for improvement is $\alpha \geq 1$. Consequently, the optimal solution for which accuracy improvement is maximized is achieved when $\tilde{\nu} = 0$. In that case, $\|\tilde{\xi}\|$ reduces to a Rayleigh distribution, which provides optimum results as seen in Fig. 5.1.

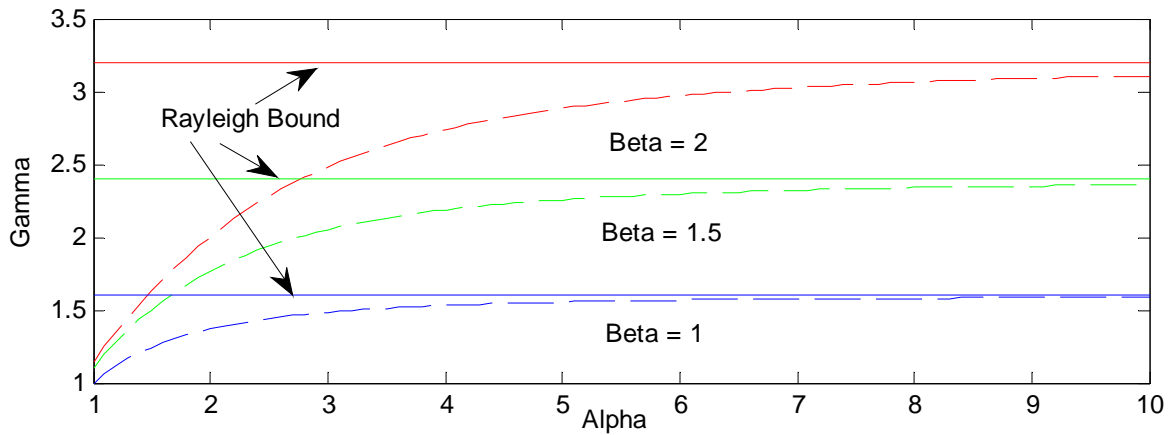


Fig. 5.1 Accuracy Improvement vs. Alpha

5.2 Inverse Estimator Weighted Average (IEWA)

The first weighted average technique is a heuristic method based on the assumption that Γ_i is a good estimator of the error. Therefore, the weights are assigned using the following inverse relation solely based on Γ_i .

$$w_i^{IE} = \frac{1/\|\mathbf{\Gamma}_i\|}{\sum_{i=1}^L 1/\|\mathbf{\Gamma}_i\|} \quad (5.8)$$

Therefore

$$\tilde{\mathbf{x}}_{IE} = \sum_{i=1}^L w_i^{IE} \hat{\mathbf{x}}_i$$

and

$$\tilde{v}_{IE} = \left\| \frac{1}{NF} \sum_{i=1}^L a_i \mathbf{u}_i + \sum_{i=1}^L w_i^{IE} \boldsymbol{\mu}_{Gi} \right\| = \|\tilde{\mathbf{r}} + \tilde{\mathbf{G}}\| \quad (5.9)$$

where,

\mathbf{u}_i : Unit vector in the direction of $\mathbf{\Gamma}_i$

$$NF = \sum_{i=1}^L 1/\|\mathbf{\Gamma}_i\|$$

$$\tilde{\mathbf{r}} = \frac{1}{NF} \sum_{i=1}^L a_i \mathbf{u}_i$$

$$\tilde{\mathbf{G}} = \sum_{i=1}^L w_i^{IE} \boldsymbol{\mu}_{Gi}$$

Now, $\|\tilde{\mathbf{r}}\|$ is a known parameter and it is bounded. The upper bound becomes very small as $L \rightarrow \infty$. In contrast, $\|\tilde{\mathbf{G}}\|$ is an unknown parameter and for higher values of $\|\tilde{\mathbf{G}}\|$, the IEWA performance degrades. Note that for a particular location, the value of $\tilde{\mathbf{G}}$ is completely dependent on the weights and can be written as follows:

$$\tilde{\mathbf{G}} = \sum_{i=1}^L w_i^{IE} \boldsymbol{\mu}_{Gi} = [\langle \boldsymbol{\omega}^{IE}, \boldsymbol{\mu}_{G_x} \rangle, \langle \boldsymbol{\omega}^{IE}, \boldsymbol{\mu}_{G_y} \rangle] \quad (5.10)$$

where,

$$\boldsymbol{\omega}^{IE} = [w_1^{IE}, w_2^{IE}, \dots, w_L^{IE}]^T$$

$$\boldsymbol{\mu}_{G_x} = [\mu_{G1_x}, \mu_{G2_x}, \dots, \mu_{GL_x}]^T$$

$$\boldsymbol{\mu}_{G_y} = [\mu_{G1_y}, \mu_{G2_y}, \dots, \mu_{GL_y}]^T$$

$\langle \cdot, \cdot \rangle$: Inner Product Operator

Therefore $\|\tilde{\mathbf{G}}\|$ and consequently \tilde{v}_{IE} increases as $\boldsymbol{\mu}_{G_x}, \boldsymbol{\mu}_{G_y}$ become co-linear with $\boldsymbol{\omega}^{IE}$.

To mitigate this issue we propose another weighted average method which will act complementary to the IEWA technique and reduce the arbitrary adverse effects of unknown parameters $\boldsymbol{\mu}_{G_x}, \boldsymbol{\mu}_{G_y}$.

5.3 Null Space Weighted Average (NSWA)

As explained in the section IV, for the maximum improvement, (*i. e.* $\min E\{\|\tilde{\xi}\|\}$)

$$\tilde{v} = \left\| \sum_{i=1}^L w_i (a_i \Gamma_i + \boldsymbol{\mu}_{Gi}) \right\| = 0 \quad (5.11)$$

$$\mathbf{A} \boldsymbol{\omega}^{NS} = \mathbf{0}$$

where

$$\mathbf{A} = [a_1 \Gamma_1 + \boldsymbol{\mu}_{G1} \quad a_2 \Gamma_2 + \boldsymbol{\mu}_{G2} \quad \dots \quad a_L \Gamma_L + \boldsymbol{\mu}_{GL}]$$

$$\boldsymbol{\omega}^{NS} = [w_1^{NS}, w_2^{NS}, \dots, w_L^{NS}]^T$$

Theoretically, this can be achieved by selecting $\boldsymbol{\omega}^{NS}$ from the null space of \mathbf{A} . However since $\boldsymbol{\mu}_G$ is unknown, we use the null space of $\mathbf{A}_D = [a_1 \Gamma_1 \quad a_2 \Gamma_2 \quad \dots \quad a_L \Gamma_L]$ to compute $\boldsymbol{\omega}^{NS}$. In addition, to compensate the adverse effects as $\boldsymbol{\mu}_{G_x}, \boldsymbol{\mu}_{G_y}$ become co-linear with $\boldsymbol{\omega}^{IE}$, we propose the following weight selection method.

Let

Ω : Null space of \mathbf{A}_D

Ψ : Null space of ω^{IE} (5.12)

Then select ω^{NS} such that

$$\omega^{NS} \in \Omega \cap \Psi \quad (5.13)$$

Therefore

$$\tilde{\mathbf{x}}_{NS} = \sum_{i=1}^L w_i^{NS} \hat{\mathbf{x}}_i \quad (5.14)$$

This selection process has two important advantages. First, by choosing ω^{NS} from the null space of \mathbf{A}_D , we force $\sum_{i=1}^L w_i^{NS} a_i \Gamma_i = \mathbf{0}$. Therefore,

$$\tilde{v}_{NS} = \left\| \sum_{i=1}^L w_i^{NS} \boldsymbol{\mu}_{G_i} \right\| = \left\| \langle \omega^{NS}, \boldsymbol{\mu}_{G_x} \rangle, \langle \omega^{NS}, \boldsymbol{\mu}_{G_y} \rangle \right\| \quad (5.15)$$

In addition, since ω^{NS} is also from the null space of ω^{IE} , NSWA works complementary to IEWA in the following sense. For given $\boldsymbol{\mu}_{G_x}, \boldsymbol{\mu}_{G_y}$

As,

$$\begin{aligned} \langle \omega^{IE}, \boldsymbol{\mu}_{G_x} \rangle, \langle \omega^{IE}, \boldsymbol{\mu}_{G_y} \rangle &\rightarrow \max \quad (\because \text{colinearity}) \\ \Leftrightarrow \langle \omega^{NS}, \boldsymbol{\mu}_{G_x} \rangle, \langle \omega^{NS}, \boldsymbol{\mu}_{G_y} \rangle &\rightarrow 0 \quad (\because \langle \omega^{NS}, \omega^{IE} \rangle \rightarrow 0) \end{aligned}$$

And, for obvious reasons the converse holds true as well.

Thus, as \tilde{v}_{IE} increases due to co-linearity, \tilde{v}_{NS} decreases. In the worst case when both $\boldsymbol{\mu}_{G_x}$ and $\boldsymbol{\mu}_{G_y}$ are co-linear with ω^{IE} ; \tilde{v}_{NS} reduces to zero and NSWA provides the optimal solution. The null spaces Ω , Ψ can be found by singular value decomposition. The vector ω^{NS} can be chosen from a linear combination of the obtained basis, and

should contain as many positive elements as possible. This can be achieved using methods such as linear programming. Since ω^{IE} consists of all positive elements, ω^{NS} will have some negative weights. The effect of these negative weights becomes negligible as L increases. The only case when the effect is noticeable is $N = 4$; where the dimension of $\Omega \cap \Psi$ reduces to 2, which is smaller than the rank of the augmented matrix $[A_D, \omega^{IE}]$. In that particular case ω^{NS} can be chosen exclusively from the null space of A_D .

5.4 Final Location Estimate

The final location estimate is an average of estimates from the IEWA and NSWA methods.

$$\tilde{\mathbf{x}}_{final} = k_{IE}\tilde{\mathbf{x}}_{IE} + k_{NS}\tilde{\mathbf{x}}_{NS} \quad (5.16)$$

where k_{IE}, k_{NS} are the level two weights ($k_{IE} + k_{NS} = 1$)

k_{IE}, k_{NS} depend on the beacon topology and they can be recessively updated, if the recent target location information is available. If no information is available, then $\tilde{\mathbf{x}}_{final}$ can be computed using simple average of $\tilde{\mathbf{x}}_{IE}, \tilde{\mathbf{x}}_{NS}$ as follows:

$$\tilde{\mathbf{x}}_{final} = \frac{\tilde{\mathbf{x}}_{IE} + \tilde{\mathbf{x}}_{NS}}{2} \quad (5.17)$$

6. RESULTS

Two approaches are compared against the proposed technique and the results are shown in the figures below. The first is the “Original” approach which considers only one combination of all available beacons. The second approach is the Rwgs method, which uses a similar weighted average algorithm proposed by Chen [1]. All simulations and experiments are performed for 20 randomly chosen target locations and results are averaged out. In the experiment, RSS to distance conversion was performed using the simplified path loss model. Fig. 6.1 shows the location error as the number of beacons increase for various mean bias values. It can be seen that the proposed technique is more stable and provides a rapid increase in accuracy compared to other approaches. Fig. 6.2 shows the probability distribution of the error for the case of $N=5$. Both the figures show that the proposed technique yields higher accuracy and robustness independent of the bias distribution. In contrast, Rwgs method is only effective for exponential bias distribution. Lastly, Fig. 6.3 shows the actual experimental results in a wireless network with $N=5$, where proposed technique performs better as well.

6.1 Simulation Results

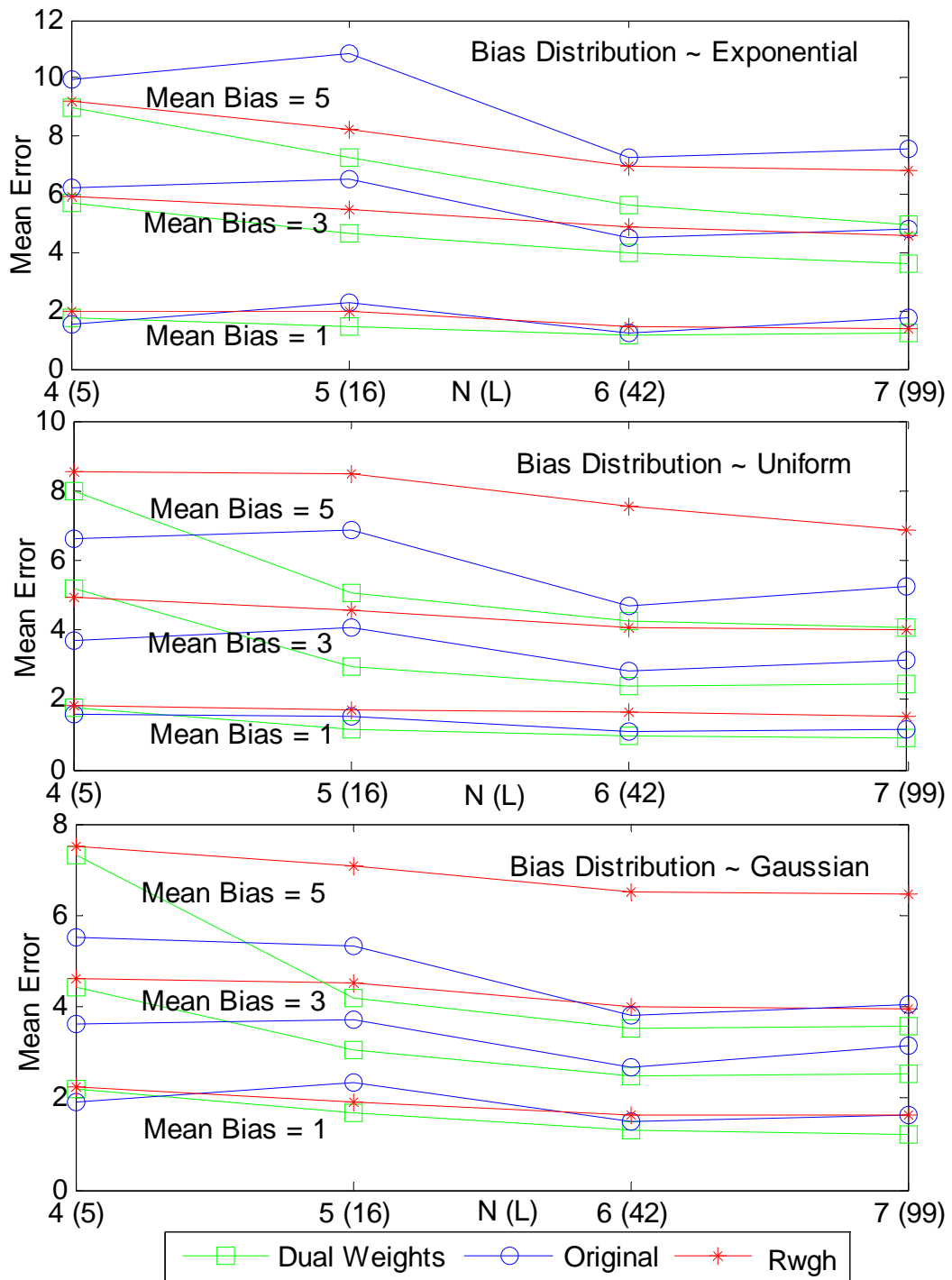


Fig. 6.1 Location Error vs. Number of Beacons

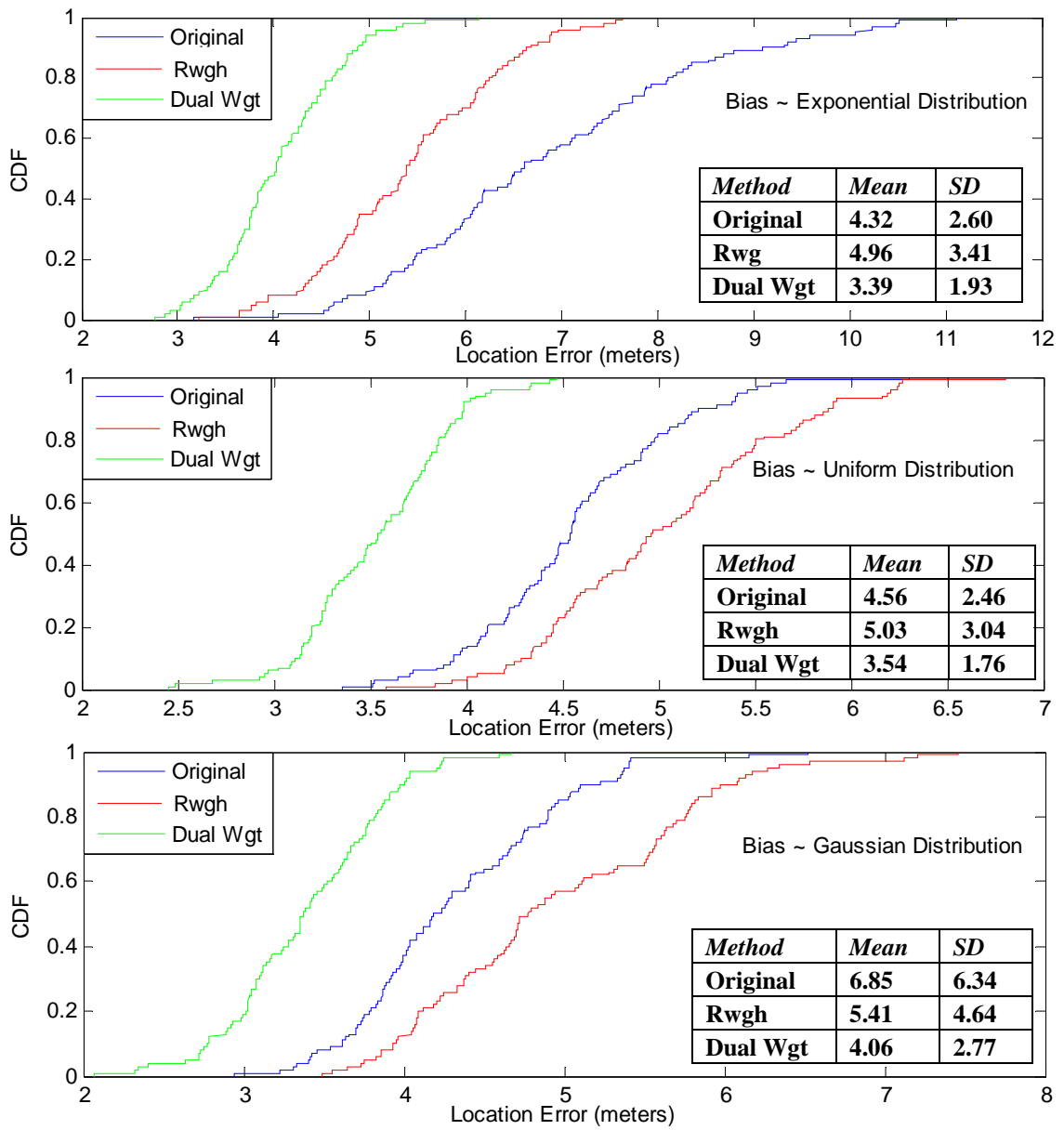


Fig. 6.2 CDF of Error for Various Bias Distributions ($\mu = 3m$)

6.2 Experimental Results

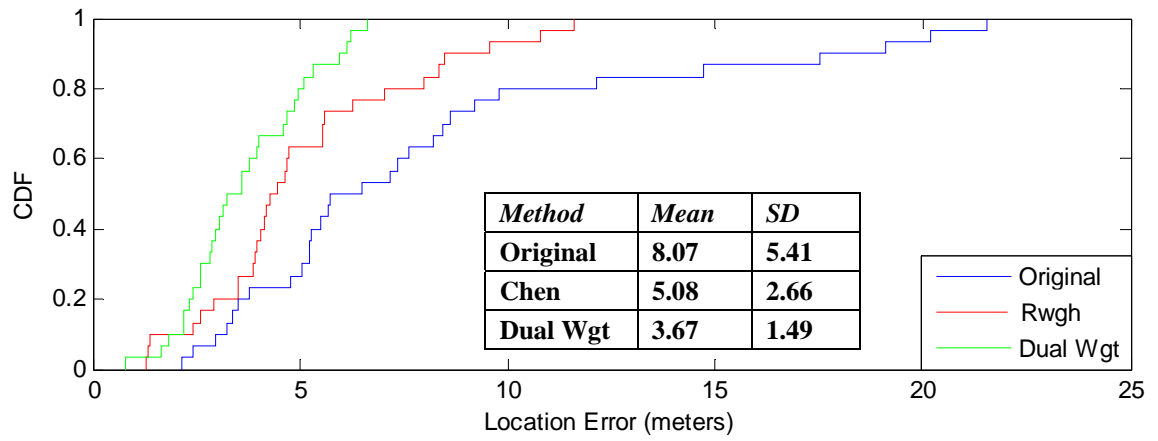


Fig. 6.3 Error Statistics from Experimental Testing

7. CONCLUSION

In this thesis, a theoretical analysis of effect of biased distance estimates on localization accuracy was provided. We also presented a dual weighted average technique to mitigate the effects of bias. The simulations show an error reduction up to 40% and 25% while the actual experiments show a reduction of 56% and 28% compared to the “Original” and Rwgh methods, respectively. A significant reduction in the variance of error is also observed. The complementary weighted average methods provide two-fold robustness, making it applicable to a wide range of scenarios independent of the bias distributions. Even though the actual experiment was carried out in a wireless network, the technique can be applied to other types of networks as well. Thus the presented technique can be used along with wide variety of localization services in a practical environment.

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CURRICULUM VITAE

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